STATEMENT REPORT

under the procedure for acquisition of the scientific degree"Doctor of Science"
of a Dsc Thesis entitled**: "Simultaneous Approximation by the Bernstein Operator",**
by candidate: **Borislav Radkov Draganov, PhD,**
In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**
Professional field: **4.5. Mathematics**
Department „ **Mathematical Analysis**"
Faculty of Mathematics and Informatics (FMI), **Sofia University "St. Kl. Ohridski" (SU),**

The statement report has been prepared by: **Prof. Dr. Stanislav Nikolaev Harizanov**, IICT-BAS, as a member of the scientific jury for the defense of this DSc Thesis according to Order № RD 38-627 /28.11.2023 of the Rector of the Sofia University.

1. **General characteristics of the dissertation thesis and the presented materials**

   The presented dissertation is written in English, contains 178 pages and consists of an introduction, 6 main chapters, and a bibliography of 100 titles, 13 of which are coauthered by the candidate (in 8 of them, he is the sole author, while each of the other 5 are written in collaboration with another scientist). The dissertation, as well as all presented materials (including extended abstracts in Bulgarian and English, CV, etc.) are very well prepared and their content correctly resembles the candidate's achievements.

2. **Short CV and personal impressions of the candidate**

   Borislav Draganov graduated from the Faculty of Mathematics and Informatics at Sofia University ``St. Kliment Ohridski" (FMI-SU) in 1998 with Master's Degree in Mathematics. In 2004 he received a PhD in Mathematics from FMI-SU under the direction of Prof. Kamen Ivanov, defending a PhD Thesis entitled „A new method for characterization of *K*-functionals and their application in Approximation Theory". Since 2022, he has been working in the department „Mathematical Analysis" at FMI-SU, where he became an Associate Professor in 2011. Since 2007, Borislav is part of the department „Mathematical Modelling and Numerical Analysis" at the Institute of Mathematics and Informatics, Bulgarian academy of Sciences.

   I have known Asc. Prof. Draganov since 2003, when we were tutoring an undergraduate course on *Differentiation and Integration* together. Since then, I have also been familiar with his research interests, having attended several of his talks on national and international scientific forums. Borislav is a very erudite person, with both deep and wide knowledge within the fields of Approximation Theory and Numerical Analysis. Meanwhile, he is a very modest and warm-hearted person. His advice helped me a lot during our joint assisstantship.

3. **Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure**

   The property of the Bernstein operators B_n to approximate not only the function, but also its derivatives (when they exist) is known in literature as *simultaneous approximation.* The main subject of the dissertation is to present estimates of the rate of this approximation. Both direct estimates and matching one- or two-term converse estimates are derived, showing that the direct estimates are sharp. The estimates are given in the ess sup-norm on the unit interval with Jacobi weights. As a further application of those results, the rates of the simultaneous approximation of the iterated Boolean sums of B_n and of two modifications of B_n, which are polynomials with integer coefficients, are characterized. Finally, the rate of convergence for

the Voronovskaya's theorem [99] is characterized in terms of *K*-functionals and moduli of smoothness. Up to my knowledge, the results in the dissertation are original. In my opinion they are interesting and may contribute to both theory and application.

4. **Approbation of the results**

The dissertation thesis is based on 8 scientific publication of Borislav Draganov ([25-33], according to the enumeration in its Bibliography, with the remark that publication [31] is a *Corrigendum* of [27], thus I do not count it independently). In the first 7 of them the candidate is the only author, while the last one is written in collaboration with Ivan Gadjev (FMI-SU). 5 of the publications are in scientific journals with impact factor, with 2 of them being in Q1, 2 of them being in Q2, and 1 being in Q4, accordiing to WoS. All publications were published after 2013, i.e., at least two years after Borislav Draganov became an Asc. Prof., which guarantees that they were not used in previous procedures by the candidate.
The results in the dissertation have been reported at numerous international scientific forums:
◆ *Conference on Harmonic Analysis and Approximation Theory*, Bellaterra, Barcelona, Spain, Jun 6-10, 2016
◆ *9th International Conference on Curves and Surfaces*, Arcachon, France, Jun 28-Jul 4, 2018 (invited talk)
◆ *IX Jaen Conference on Approximation Theory*, Ubeda, Spain, Jul 8-13, 2018
◆ *First Analysis Mathematica International Conference*, Budapest, Hungary, Aug 12-17, 2019
◆ International Online Workshop on Approximation Theory, Ivano-Frankivsk, Ukraine, Mar 19-21, 2021 (invited plenary talk)
There are 21 independent citations of Borislav Draganov for the period 2020-2024 (according to SCOPUS).
All mentioned above is an indication not only for the relevance and excellence of the conducted research but also for the international recognizability of Borislav Draganov within the scientific field. Finally, it is important to be noted that
a) the scientific works **meet** the minimum national requirements (under Art. 2b, para. 2 and 3 of ADASRB*) and respectively to the additional requirements of Sofia University "St. Kliment Ohridski" for acquiring the scientific degree "Doctor of Science" in the scientific field and professional field of the procedure;
b) the results presented by the candidate in the dissertation work and scientific works, related to it **do not repeat** such from previous procedures for acquiring a scientific title and academic position;
c) there is **no** plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure.

5. **Qualities of the abstract**

The two abstracts, submitted by the applicant, are written in Bulgarian and English and their length is 36, respectively 35 pages. Both abstracts meet all the requirements for their preparation and correctly present the results and the content of the dissertation.

6. **Critical notes and recommendations**

I have no critical remarks, which could question the contributions of the applicant and which would affect my overall positive assessment on the dissertation thesis.

7. **Conclusion**

Having become acquainted with the DSc Thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented DSc Thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of the scientific degree "Doctor of Science" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I strongly recommend** the scientific jury to award Dr. Borislav Radkov Draganov, the scientific degree „Doctor of Science" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics (Mathematical Analysis).

Date: 23.02.2024 Signature: .....

/Prof. Stanislav Harizanov, PhD/